Running of the Spectral Index in Noncommutative Inflation

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Abstract: We study the cosmological implications of the space-space noncommutative inflation and present formulae for the spectral index and its running. Our results show that deviations from the spectral index and its running depend on the space-space noncommutativity length scale. We conclude that in the slow-roll regime of a typical inflationary scenario, space-space noncommutativity has negligible effects on both the spectral index and its running. Two classes of examples have been studied and comparisons made with the standard slow-roll formulae. The results show that the correction terms to the commutative case are positive for the spectral index and negative for the running of the spectral index. Nevertheless the spectral index in noncommutative spaces is less than one owing to the very small values of the correction terms.

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1. Introduction

It is generally believed that the picture of space-time as a manifold should break down at very short distances of the order of the Planck length. Field theories on noncommutative spaces may play an important role in unraveling the properties of nature at the Planck scale. It has been shown that the noncommutative geometry naturally appears in string theory with a non zero antisymmetric B-field [1].

Besides the string theory arguments the noncommutative field theories by themselves are very interesting. In a noncommutative space-time the coordinates operators satisfy the commutative relation

$$[\hat{x}^\mu, \hat{x}^\nu] = i\theta^{\mu\nu}, \tag{1.1}$$

where $\hat{x}$ are the coordinate operators and $\theta^{\mu\nu}$ is an antisymmetric tensor of dimension of $(\text{length})^2$. Generally noncommutative version of a field theory is obtained by replacing the product of the fields appearing in the action by the star products

$$(f \star g)(x) = \exp\left(\frac{i}{2}\theta^{\mu\nu}\frac{\partial}{\partial x^\mu}\frac{\partial}{\partial y^\nu}\right) f(x)g(y) \mid_{x=y} \tag{1.2}$$

where f and g are two arbitrary functions which we assume to be infinitely differentiable.

In recent years there have been a lot of work devoted to the study of noncommutative field theory or noncommutative quantum mechanics and possible experimental consequences of extensions of the standard formalism [2-13]. In the last few years there has been also a growing interest in probing the space-space noncommutativity effects on cosmological observations [14-26].

Apart from the field theory or quantum mechanics, we are more interested in some possible cosmological consequences of noncommutativity in space. In so doing, we study the effects of space-space noncommutativity on the spectral index and its running. This issue has been studied in the last few years by several authors [21-26].

The theory of inflation [27] has faired well in this latest round of cosmological discoveries. Generically, slow-roll inflation predicts the Universe is flat, and that the primordial perturbations are Gaussian, adiabatic, and have a nearly scale invariant spectrum. The degree to which slow-roll inflation predicts a scale invariant spectrum depends on the dynamics of the scalar field(s) controlling inflation. The simplest possibility is a single 'inflaton', slowing rolling down its potential with its kinetic energy strongly damped by the Hubble expansion. In the limit in which the rolling is infinitely slow and the damping infinitely strong, the primordial spectrum is a power law, with index n exactly equal to one. Deviations from $n = 1$ are measures of how slowly the field rolled and how strongly its motion was damped during inflation. Equivalently, different inflationary models predict different values of n or more generally of the shape of the spectrum, measurements of this primordial spectrum enable one to discriminate among different inflationary models.

There is another reason why precise measurements of the primordial spectrum are important to proponents of inflation. Even before inflation was proposed, Harrison and Zel'dovich introduced the notion that scale free ($n = 1$) adiabatic perturbations represent natural initial conditions. A spectrum with n not exactly equal to one, or even more telling, one with deviations from a pure power law form, is perfectly compatible with inflation. While not a *proof* of inflation, such deviations would surely be a *disproof* of the Harrison-Zel'dovich speculations.

The spectral index and its running in commutative geometry have been studied by several authors, for example [28-37]. In the slow-roll approximation, $|n-1|$ is small. It is often assumed [30] that deviations from a pure power law are of order of $(n-1)^2$. If true, this would mean that the recent measurements indicating $|n-1|$ is smaller than about 0.1 imply that deviations from a power law would only show up at the percent level at best.

We will use a natural unit system that sets k_B, c and $\hbar$ all equal to 1, so that $\ell_P = M_P^{-1} = \sqrt{G}$.

The plan of this article is as follows. In section 2, we give a brief review of the

spectral index and its running in the standard and commutative inflation. In section 3, we first present the explicit and general formulae of the spectral index and its running in noncommutative spaces and then apply them to two classes of examples of the inflaton potential. Finally, we discuss our results and conclude in section 4.

2. Running of the spectral index in commutative inflation

In general, the power spectrum of the scalar perturbations is closely related to the functional form of the inflaton potential, $V(\phi)$ [37]

$$\delta_H^2 = \frac{128\pi}{3M_P^6}\frac{V^3}{V'^2}, \tag{2.1}$$

where M_P is the Planck mass and a prime denotes $d/d\phi$[1]. The relationship between the inflaton field and comoving wave-number follows from the scalar field equations of motion and is given by

$$\frac{d}{d\ln k} = -\frac{M_P^2}{8\pi}\frac{V'}{V}\frac{d}{d\phi} \tag{2.2}$$

in the slow-roll limit. By defining the slow-roll parameters as [29]

$$\epsilon \equiv \frac{M_P^2}{16\pi}\left(\frac{V'}{V}\right)^2, \tag{2.3}$$

$$\eta \equiv \frac{M_P^2}{8\pi}\frac{V''}{V}, \tag{2.4}$$

$$\xi \equiv \frac{M_P^4}{64\pi^2}\frac{V'V'''}{V^2}, \tag{2.5}$$

the spectral index and its running may be expressed directly in terms of the potential and its derivatives [28,30]

$$n_{\rm S} - 1 = \frac{d\ln\delta_H^2}{d\ln k} = -6\epsilon + 2\eta, \tag{2.6}$$

$$\frac{dn_{\rm S}}{d\ln k} = 16\epsilon\eta - 24\epsilon^2 - 2\xi. \tag{2.7}$$

Then the running of the spectral index depends on the third derivative of the potential. Eq. (2.7) is truncated at order ξ, such that quadratic corrections in ϵ and η are assumed to be negligible. This requires that $|\xi| \ll \max(\epsilon, |\eta|)$ and is equivalent to assuming that $|n_S - 1| \ll 1$ and $|dn_S/d\ln k| \approx (n_S-1)^2$ or less. As emphasized in Refs. [34,35], slow-roll predicts the former condition but not necessarily the latter.

[1]We employ the normalization conventions of Ref.[31].

3. Running and space-space noncommutativity

For a polynomial potential the *effective* action will be of the form [22]

$$S = \int d^4x\sqrt{-g}\Big[\frac{1}{2}(\partial_\mu\phi)\star(\partial^\mu\phi) - \frac{\lambda}{n!}\phi\star ...\star\phi\Big]$$
$$\equiv \int d^4x\sqrt{-g}\Big[\frac{1}{2}(\partial_\mu\phi)\star(\partial^\mu\phi) - \frac{\lambda}{n!}\phi^{\star n}\Big]. \tag{3.1}$$

With no loss of generality, the authors of Ref. [22] choose a frame in which the only nonvanishing space-space component of $\theta^{\mu\nu}$ is

$$\theta^{12}(t) = \frac{1}{\Lambda^2 a^2}, \tag{3.2}$$

where a is the scale factor of the Universe and Λ^{-1} the noncommutativity length scale. Using ϑ as the angle between the comoving wave-number k and the third axis, the scalar fluctuations have the late time amplitude [22]

$$|\phi_{\rm k}| = \frac{H}{\sqrt{2}k^{3/2}}\left(1 - \frac{3}{32}\frac{H^4}{\Lambda^4}\sin^2\vartheta\right). \tag{3.3}$$

The first term in (3.3) is the standard result, valid for fluctuations of a massless field in a de Sitter Universe, see Ref. [38]. The second is instead a new effect induced by $\theta^{\mu\nu}$ depending terms in the action (3.1), and explicitly shows the presence of a preferred direction associated with the nonvanishing component θ^{12}. It is worth noticing that this correction does not decrease at later times, so that the imprint of noncommutativity is preserved even after the physical scales of the fluctuations have grown to much larger sizes than Λ^{-1}. Our interested purpose here is actually determination of both the spectral index and its running in space-space noncommutativity, but this was not done in [22].

In the commutative or usual inflation, the density perturbations can be expressed by [38]

$$\delta_H^2 = \frac{4\pi k^3}{(2\pi)^3}|\phi_k|^2\left(\frac{H}{\dot\phi}\right)^2. \tag{3.4}$$

Using (3.3) and (3.4), we have

$$\delta_H^2 = \left(\frac{H}{2\pi}\right)^2\left(1 - \frac{3}{32}\frac{H^4}{\Lambda^4}\sin^2\vartheta\right)^2\left(\frac{H}{\dot\phi}\right)^2. \tag{3.5}$$

Based on the slow-roll approximation, the equations of motion take the forms [38]

$$H^2 = \frac{8\pi}{3M_P^2}V, \tag{3.6}$$

$$3H\dot\phi = -V', \tag{3.7}$$

where we assume the Universe to be spatially flat and the inflaton field ϕ to be spatially homogeneous. From (3.6) and (3.7), for space-space noncommutative geometry the density perturbation is

$$\delta_H^2 = \frac{128\pi}{3M_P^6}\frac{V^3}{V'^2}\left(1-\frac{3}{32}\frac{H^4}{\Lambda^4}\sin^2\vartheta\right)^2. \tag{3.8}$$

When $\Lambda \to +\infty$ or $\Lambda^{-1} = 0$, Eq.(3.8) reproduces the standard and commutative density perturbation. Substituting (3.8) in (2.6) and (2.7), one can easily obtain the formulae for the spectral index and its running

$$n_{\rm S} - 1 = -6\epsilon + 2\eta + \frac{\pi\sin^2\vartheta}{3M_P^2\Lambda^4}V'^2\left(1-\frac{3}{32}\frac{H^4}{\Lambda^4}\sin^2\vartheta\right)^{-1}, \tag{3.9}$$

$$\begin{aligned}\frac{dn_{\rm S}}{d\ln k} &= 16\epsilon\eta - 24\epsilon^2 - 2\xi - \frac{\sin^2\vartheta}{3\Lambda^4}\left(1-\frac{3}{32}\frac{H^4}{\Lambda^4}\sin^2\vartheta\right)^{-2}\\ &\times\left[\frac{V'^2V''}{4V}\left(1-\frac{3}{32}\frac{H^4}{\Lambda^4}\sin^2\vartheta\right)+\frac{\pi^2V'^4\sin^2\vartheta}{6M_P^4\Lambda^4}\right].\end{aligned} \tag{3.10}$$

The point is that the correction term to the spectral index which is the third term of the RHS of (3.9) and to the running of the spectral index which is the fourth term of the RHS of (3.10) have positive and negative values, respectively. The sum of the first and second term in the RHS of (3.9) is negative[2]. This makes $n_S - 1$ to have negative value and so n_S to be a few smaller than one in commutative inflation. Due to very small value of Λ^{-1}, being of the order of the Planck length, the correction terms to the commutative spectral index is much smaller than $-6\epsilon+2\eta$, see Eq.(3.9). Consequently, the value of $n_S - 1$ is negative and so n_S is a few smaller than one in noncommutative inflation. Let us now use (3.9) and (3.10) for two potentials of the inflation. We first study $V(\phi) = m^2\phi^2/2$ and then $V(\phi) = \lambda\phi^4$. In the limit of $\Lambda \to +\infty$, (3.9) and (3.10) approach to (2.6) and (2.7), respectively.

3.1 The first example

For the purpose of illustration, we now consider the potential $V(\phi) = m^2\phi^2/2$

$$\epsilon = \eta = \frac{M_P^2}{4\pi\phi^2}, \tag{3.11}$$

$$\xi = 0. \tag{3.12}$$

[2]See the first term of the RHS of (3.17) and (3.27) which have negative values.

Using the definition of e-folding in inflation [38]

$$N = -\frac{8\pi}{M_P^2}\int_{\phi}^{\phi_{\rm f}} \frac{V}{V'}d\phi, \tag{3.13}$$

we have

$$N = \frac{2\pi}{M_P^2}(\phi^2 - \phi_{\rm f}^2), \tag{3.14}$$

where $\phi_{\rm f}$ is the value of the inflaton field at the end of inflation. To obtain e-folding as a function of the inflaton field, we must obtain $\phi_{\rm f}$. From $\epsilon = \eta = 1$ we get

$$\phi_{\rm f} = \frac{M_P}{\sqrt{4\pi}}. \tag{3.15}$$

From (3.14) and (3.15), we have

$$\phi^2 = \frac{M_P^2}{4\pi}(2N+1). \tag{3.16}$$

With no loss of generality, we take $\vartheta = \frac{\pi}{2}$. So substituting $\sin\vartheta = 1$ in the above equations leads us to

$$n_{\rm S} - 1 = -\frac{4}{(2N+1)} + \frac{(2N+1)m^4}{12\Lambda^4}\left[1 - \frac{(2N+1)^2 m^4}{96\Lambda^4}\right]^{-1}, \tag{3.17}$$

$$\frac{dn_{\rm S}}{d\ln k} = -\frac{8}{(2N+1)^2} - \frac{m^4}{6\Lambda^4}\left(1 - \frac{(2N+1)^2 m^4}{96\Lambda^4}\right)^{-2}\left[1 + \frac{(2N+1)^2 m^4}{96\Lambda^4}\right]. \tag{3.18}$$

The point is that the general formulae of (3.17) and (3.18) in terms of unknown value of ϑ could be obtained by substituting Λ^2 with $\Lambda^2/\sin\vartheta$, see (3.3).

We know the mass of the inflaton field to be $m \simeq 1.21 \times 10^{-6} M_P$, so the term of $\frac{m^4(2N+1)^2}{96\Lambda^4}$ is much less than one and we can rewrite (3.17) and (3.18) up to the second order of m^4/Λ^4 for the spectral index

$$\begin{aligned} n_{\rm S} - 1 &\simeq -\frac{4}{(2N+1)} + \frac{(2N+1)m^4}{12\Lambda^4}\left[1 + \frac{(2N+1)^2 m^4}{96\Lambda^4}\right] + ... \\ &\simeq -\frac{4}{(2N+1)} + \frac{(2N+1)m^4}{12\Lambda^4} + \frac{(2N+1)^3 m^8}{1152\Lambda^8} + ..., \end{aligned} \tag{3.19}$$

and up to the third order of m^4/Λ^4 for the running of the spectral index

$$\begin{aligned} \frac{dn_{\rm S}}{d\ln k} &\simeq -\frac{8}{(2N+1)^2} - \frac{m^4}{6\Lambda^4}\left(1 + \frac{(2N+1)^2 m^4}{48\Lambda^4}\right)\left[1 + \frac{(2N+1)^2 m^4}{96\Lambda^4}\right] + ... \\ &\simeq -\frac{8}{(2N+1)^2} - \frac{m^4}{6\Lambda^4} - \frac{(2N+1)^2 m^8}{192\Lambda^8} - \frac{(2N+1)^4 m^{12}}{27648\Lambda^{12}} + ... \end{aligned} \tag{3.20}$$

These equations tell us that the effects of the space-space noncommutativity on the spectral index and its running are of the order of m^4/Λ^4, m^8/Λ^8 and m^{12}/Λ^{12} or so. According to the value of e-folding to be approximately 60 for solving the problems of the standard cosmology, and taking Λ^{-1} to be of the order of the Planck length, we conclude that the second, third and fourth terms in the RHS of (3.19) and (3.20) are much smaller than the first term because $m/M_P \sim 10^{-6}$. Therefore $m^4/\Lambda^4 \sim 10^{-24}$, $m^8/\Lambda^8 \sim 10^{-48}$ and $m^{12}/\Lambda^{12} \sim 10^{-72}$. Since the accuracy of the Wilkinson Microwave Anisotropy Probe (WMAP) data of the spectral index and its running is up to only two or three decimal integers, for example $n_S = 0.99 \pm 0.04$ [39] or $dn_S/d\ln k = -0.055^{+0.028}_{-0.029}$ [40], we conclude that the effects of the noncommutativity of spaces which are of the order of m^4/Λ^4 and its powers of 2 and 3 or so cannot be currently detected by the present experimental celestial or ground instruments. If true, the imprints of the space-space noncommutative geometry on the spectral index and its running are undetectable and negligible.

3.2 The second example

For the second example, we study $V(\phi) = \lambda\phi^4$. The slow-roll parameters are

$$\epsilon = \frac{M_P^2}{\pi\phi^2}, \tag{3.21}$$

$$\eta = \frac{3M_P^2}{2\pi\phi^2}, \tag{3.22}$$

$$\xi = \frac{3M_P^4}{2\pi^2\phi^4}. \tag{3.23}$$

Using the definition of e-folding in inflation, we have

$$N = \frac{\pi}{M_P^2}\left(\phi^2 - \phi_{\mathrm{f}}^2\right). \tag{3.24}$$

To obtain the e-folding number as a function of the inflaton field, we must obtain the value of the inflaton field at the end of inflation, ϕ_{f}. From $\epsilon = 1$ we get

$$\phi_{\mathrm{f}} = \frac{M_P}{\sqrt{\pi}}. \tag{3.25}$$

Assuming $\eta = 1$ and $\xi = 1$, we obtain $\phi_{\mathrm{f}} = \sqrt{\frac{3}{2\pi}}M_P$ and $\phi_{\mathrm{f}} = (\frac{3}{2\pi^2})^{1/4}M_P$, respectively. These values of ϕ_{f} based on the conditions $\eta = 1$ and $\xi = 1$ are larger than ϕ_{f} arisen from $\epsilon = 1$, see (3.25). We here take the condition $\epsilon = 1$ by itself is a true condition to obtain ϕ_{f}. From (3.24) and (3.25) we have

$$\phi^2 = \frac{M_P^2(N+1)}{\pi}. \tag{3.26}$$

Again we take $\vartheta = \frac{\pi}{2}$. Substituting $\sin\vartheta = 1$ in (3.9) and (3.10) and using (3.21), (3.22) and (3.23) we obtain the main formulae for the spectral index and its running

$$n_{\rm S} - 1 = -\frac{3}{(N+1)} + \frac{16\lambda^2 M_P^4}{3\pi^2\Lambda^4}(N+1)^3\Big[1 - \frac{2\lambda^2 M_P^4 (N+1)^4}{3\pi^2\Lambda^4}\Big]^{-1}, \tag{3.27}$$

$$\frac{dn_{\rm S}}{d\ln k} = -\frac{3}{(N+1)^2} - \frac{16\lambda^2 M_P^4 (N+1)^2}{3\pi^2\Lambda^4}\Big(1 - \frac{2\lambda^2 M_P^4 (N+1)^4}{3\pi^2\Lambda^4}\Big)^{-2} \times \Big[3 + \frac{2\lambda^2 M_P^4 (N+1)^4}{3\pi^2\Lambda^4}\Big]. \tag{3.28}$$

Considering the noncommutativity length scale Λ^{-1} to be of the order of the Planck scale, so we obtain M_P^4/Λ^4 of the order of unity, $\mathcal{O}(1)$. Potentials in chaotic inflation are characterized by a small overall coupling constant, $\lambda \simeq 10^{-15}$, to ensure consistency with the amplitude of energy density perturbations around 10^{-5}, for ϕ field values of a few Planck units. So we can expand (3.27) and (3.28) in terms of $\frac{2\lambda^2 M_P^4 (N+1)^4}{3\pi^2\Lambda^4}$. After a lengthy but straightforward calculation, we find

$$\begin{aligned} n_{\rm S} - 1 &\simeq -\frac{3}{(N+1)} + \frac{16\lambda^2 M_P^4}{3\pi^2\Lambda^4}(N+1)^3\Big[1 + \frac{2\lambda^2 M_P^4 (N+1)^4}{3\pi^2\Lambda^4}\Big] + ... \\ &\simeq -\frac{3}{(N+1)} + \frac{16\lambda^2 M_P^4 (N+1)^3}{3\pi^2\Lambda^4} + \frac{32\lambda^4 M_P^8 (N+1)^7}{9\pi^4\Lambda^8} + ..., \end{aligned} \tag{3.29}$$

$$\begin{aligned} \frac{dn_{\rm S}}{d\ln k} &= -\frac{3}{(N+1)^2} - \frac{16\lambda^2 M_P^4 (N+1)^2}{3\pi^2\Lambda^4}\Big(1 + \frac{4\lambda^2 M_P^4 (N+1)^4}{3\pi^2\Lambda^4}\Big) \\ &\quad \times \Big[3 + \frac{2\lambda^2 M_P^4 (N+1)^4}{3\pi^2\Lambda^4}\Big] + ... \\ &\simeq -\frac{3}{(N+1)^2} - \frac{48\lambda^2 M_P^4 (N+1)^2}{3\pi^2\Lambda^4} - \frac{224\lambda^4 M_P^8 (N+1)^6}{9\pi^4\Lambda^8} \\ &\quad - \frac{128\lambda^6 M_P^{12} (N+1)^{10}}{27\pi^6\Lambda^{12}} + ..., \end{aligned} \tag{3.30}$$

where we have truncated at the second order of $\lambda^2 M_P^4/\Lambda^4$ for the spectral index, and also at the third order of $\lambda^2 M_P^4/\Lambda^4$ for the running of the spectral index. Again similar to the case of $m^2\phi^2/2$ we conclude that in comparison with the first terms in the RHS of (3.29) and (3.30), the second, third and fourth terms in RHS of mentioned equations can be ignored because $\lambda \sim 10^{-15}$ and the e-folding number to be approximately close to 60. So, the imprints of the space-space noncommutative geometry are negligible by considering the accuracy of the WMAP data and of the present experimental celectial or ground instruments.

4. Conclusions

We have studied the effects of the space-space noncommutative geometry on the spectral index and its running, and obtained the corrections arisen from the space-space noncommutativity. If there exists any noncommutativity of space in nature, as it seems to emerge from different theories and arguments, its implications should appear in the spectral index and its running. We presented the general features of our formalism and applied it to two specific potentials of chaotic inflation. The results showed that the terms arising from the noncommutativity of space depend on the space-space noncommutativity length scale.

For the purpose of illustration, we present the formulae for the spectral index and its running in noncommutative inflation for two classes of examples of the inflationary potentials. The first one is $m^2\phi^2/2$ and the second $\lambda\phi^4$. For the former we conclude that the space-space noncommutativity effects on both the spectral index and its running to be of the order of m^4/Λ^4 and its powers of 2, 3 or so, and for the latter to be of the order of $\lambda^2 M_P^4/\Lambda^4$ and its power of 2, 3 or so. Our results are in terms of the noncommutativity length scale Λ^{-1}, to be approximately close to the Planck length. In the limit of $\Lambda \to +\infty$ or $\Lambda^{-1} = 0$, our formalism approach to those of the usual or commutative inflation [38].

Since the values of m^4/Λ^4 and $\lambda^2 M_P^4/\Lambda^4$ are very small, the effects of space-space noncommutativity on the spectral index and its running are very small. Considering the accuracy of the WMAP data which is up to two or three decimal integers as given in [39] and [40], we conclude that the effects of the space-space noncommutative geometry on the spectral index and its running cannot be currently detected. This means that the noncommutativity corrections, being of the order of 10^{-24} or so for $V(\phi) = m^2\phi^2/2$ and 10^{-30} or so for $V(\phi) = \lambda\phi^4$, are both negligible and undetectable by the present experimental celectial or ground instruments. Similar to our results here, recent paper [19] shows that in the slow-roll regime of a typical chaotic inflationary scenario, noncommutativity of space has negligible impact.

In general, commutative inflationary cosmology satisfying the slow-roll conditions naturally predicts positive spectral index which is a few smaller than one and also predicts negative and very small value for the running of the spectral index. Our results as given in (3.9) and (3.10) show that the correction terms to the commutative case are positive for $n_S - 1$ and negative for the running of the spectral index. This means that the space-space noncommutativity causes a negligible increasing in the value of $n_S - 1$ and a negligible decreasing in the value of $\frac{dn_S}{d\ln k}$. Nevertheless the spectral index in the space-space noncommutative inflation is less than one owing to the very small values of the correction terms. This result for the value of the spectral index in noncommutative

spaces is in agreement with the WMAP data, for example $n_S = 0.99 \pm 0.04$ [39].

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